SINP-TNP/05-07

Asymptotic Quasinormal Modes of d Dimensional Schwarzschild Black Hole with Gauss-Bonnet Correction

Sayan K. Chakrabarti[1] and Kumar S. Gupta[2]

Theory Division
Saha Institute of Nuclear Physics
1/AF Bidhannagar, Calcutta - 700064, India

Abstract

We obtain an analytic expression for the highly damped asymptotic quasinormal mode frequencies of the $d \geq 5$-dimensional Schwarzschild black hole modified by the Gauss-Bonnet term, which appears in string derived models of gravity. The analytic expression is obtained under the string inspired assumption that there exists a mimimum length scale in the system and in the limit when the coupling in front of the Gauss-Bonnet term in the action is small. Although there are several similarities of this geometry with that of the Schwarzschild black hole, the asymptotic quasinormal mode frequencies are quite different. In particular, the real part of the asymptotic quasinormal frequencies for this class of single horizon black holes in not proportional to log(3).

October 2005

PACS : 04.70.-s

[1]Email: sayan@theory.saha.ernet.in
[2]Email: kumars.gupta@saha.ac.in (Corresponding author)

1. Introduction

Quasinormal modes associated with the perturbations of a black hole metric in classical general relativity have been found to be a useful probe of the underlying space-time geometry [1, 2]. The quasinormal ringing frequencies carry unique information about black hole parameters and they are expected to be detected in the future gravitational wave detectors [3] and possibly even in the Large Hadron Collider [4]. For asymptotically flat space-times, these metric perturbations are solutions of the corresponding wave equation with complex frequencies which are purely ingoing at the horizon and purely outgoing at infinity [5]. For most geometries, the wave equation is not exactly solvable and various schemes have been used in the literature to obtain approximate analytical expression for the quasinormal modes. In particular, Motl and Neitzke [6] have proposed a geometric method for calculating the highly damped quasinormal modes in asymptotically flat geometries. Their method involves the extension of the wave equation beyond the physical region between the horizon and infinity by analytically continuing the radial variable r to the whole complex plane. The asymptotic quasinormal modes are then obtained using suitable monodromy relations in the complex plane that encode the appropriate boundary conditions. This monodromy approach has subsequently been applied to more general geometries [7, 8, 9, 10] and has also been extended to the study of "non-quasinormal modes" of various black holes as well [11, 12].

In spite of their classical origin, it has recently been proposed that the quasinormal modes might provide a glimpse into the quantum nature of black holes. One such proposal by Hod [13] is associated with the idea that black holes have a discrete area spectrum [14], with the area being quantized in integer multiples of $4\log(k)$, where $k > 1$ is an undetermined positive integer. Hod's conjecture is based on the observation that the real part of the asymptotic highly damped quasinormal modes of the Schwarzschild black hole is independent of space-time dimensions as well as the nature of the metric perturbation and proportional to $\log(3)$. This universality strongly suggests that the real part of the highly damped asymptotic quasinormal frequency of the Schwarzschild black hole is a characteristic feature of the black hole itself. This observation together with the area quantization law and first law of black hole mechanics immediately leads to the conclusion that $k = 3$. However, for geometries other than Schwarzschild, the validity of Hod's conjecture is debatable [15, 7, 9]. In a related work, it was shown that the real part of the asymptotic quasinormal mode of the Schwarzschild black hole can be used to determine the Immirzi parameter appearing in loop quantum gravity [16]. Independent of these connections, it has also been shown that the quasinormal modes for various background geometries appear naturally in the description of the corresponding dual CFT's living on the black hole horizons [17]. These results have mostly been obtained for geometries which are solutions of the equation of motion arising from the classical Einstein-Hilbert action.

In this Paper we shall analyze the asymptotic quasinormal modes of the d dimensional Schwarzschild black holes in presence of a Gauss-Bonnet correction term[18, 19], which appear when the Einstein-Hilbert action is generalized to include the leading order higher curvature terms arising from the low energy limit of string theories [20]. There has been a renewed interest in Gauss-Bonnet black holes in the context of brane-world models [21] and the entropy and

related thermodynamic properties of such black holes have been discussed in recent literature [22]. The quasinormal frequencies of certain low lying modes of the Gauss-Bonnet black hole have also been estimated in the WKB approximation [23]. Our interest is in the other end of the quasinormal frequency spectrum, namely in the highly damped asymptotic regime. In addition, we want to study the properties of asymptotic quasinormal modes when the geometry is minimally different from the Schwarzschild background. In order to make this precise, we shall assume that there exists a fundamental minimum length scale in the system, in terms of which the smallness of the Gauss-Bonnet coupling would be specified. This assumption is justified by the fact that the Gauss-Bonnet term has its natural origin within the frameworks of string theory, which in turn postulates the existence of a fundamental minimum length scale in nature. In our analysis we shall use this string inspired assumption although the details of such a length scale would not be important. In the limit of a small Gauss-Bonnet coupling, the resulting classical geometry is very similar to Schwarzschild case. We shall however show that there are important differences in the asymptotic form of the quasinormal modes. In particular, the real part of the asymptotic quasinormal mode is dimension dependent and is not proportional to log3. Our result therefore encodes the effect of a small Gauss-Bonnet term on the asymptotic quasinormal modes of the d dimensional Schwarzschild black hole.

This Paper is organized as follows. In Section 2 we shall briefly review the Gauss-Bonnet term and discuss the modification of the d dimensional Schwarzschild metric in presence of a weakly coupled Gauss-Bonnet term in the action. In Section 3 we shall apply the monodromy method to obtain the asymptotic quasinormal modes of this system. Section 4 concludes the paper with some discussions of our result and an outlook.

2. The d-dimensional Schwarzschild metric with the Gauss-Bonnet term in the weak coupling limit

In this Section we shall discuss some properties of the d-dimensional Schwarzschild metric due to a Gauss-Bonnet term in the limit of small Gauss-Bonnet coupling. In space-time dimensions $d \geq 5$, the Einstein-Hilbert action in presence of the Gauss-Bonnet term has the form

$$I = \frac{1}{16\pi}\left[\int d^dx\sqrt{-g}R + \frac{\alpha}{(d-3)(d-4)}\int d^dx\sqrt{-g}(R_{abcd}R^{abcd} - 4R_{cd}R^{cd} + R^2)\right], \tag{2.1}$$

where we have set the Newton's constant G_d in d space-time dimensions and the velocity of light c equal to one. The parameter α in Eqn. (2.1)is the Gauss-Bonnet coupling. We shall consider only positive α, which is consistent with the string expansion [18].

The field equations can be written as:

$$\delta I/\delta g_{\mu\nu} = -G_{\mu\nu} + \alpha T_{\mu\nu} = 0 \tag{2.2}$$

where

$$T_{\mu\nu} = RR_{\mu\nu} - R_{\mu\alpha\beta\gamma}R_{\nu}{}^{\alpha\beta\gamma} - 2R_{\alpha\beta}R^{\alpha}{}_{\mu}{}^{\beta}{}_{\nu} - 2R_{\mu\alpha}R^{\alpha}{}_{\nu} - \frac{1}{4}(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2). \tag{2.3}$$

When $\alpha = 0$, the solution of the equations of motion is given by the Schwarzschild metric. For small values of α, the second term in Eqn. (2.2) would provide corrections to the Schwarzschild geometry.

The metric for the spherically symmetric, asymptotically flat black hole solution of mass M arising from the action in Eqn. (2.1) is given by

$$ds^2 = -f(r)dt^2 + f^{-1}dr^2 + r^2 d\Omega_{d-2}^2, \tag{2.4}$$

$$f(r) = 1 + \frac{r^2}{2\alpha} - \frac{r^2}{2\alpha}\sqrt{1 + \frac{8\alpha M}{r^{d-1}}}, \tag{2.5}$$

where r is the radial variable in d space-time dimensions and Ω_{d-2}^2 is the metric on the $(d-2)$ dimensional sphere. Eqns. (2.4)-(2.5) give an exact solution to the equations of motion arising from the action I in (2.1). However, as mentioned before, we are interested in small deviations from the Schwarzschild geometry, which is obtained for a small value of α. Thus, in the limit $\alpha \to 0$, the function $f(r)$ in the metric (2.4) is given by

$$f(r) = 1 - \frac{2M}{r^{d-3}} + \frac{4\alpha M^2}{r^{2d-4}} \tag{2.6}$$

The series expansion of Eqn. (2.5) can be done if $\frac{8\alpha M}{r^{d-1}} << 1$, which naturally leads to the question that is such an expansion valid for our analysis. In order to address this issue, note that the Gauss-Bonnet term has its natural origin in the framework of string theory which postulates the existence of a fundamental minimum length scale l, which is finite but can be arbitrarily small. The physics of any distance below this scale simply cannot be described in this context. In this scenario, if we choose α such that it satisfies the relation $\frac{8\alpha M}{l^{d-1}} << 1$, then the relation $\frac{8\alpha M}{r^{d-1}} << 1$ is satisfied for all $r > l$, i.e. for all distances that can be physically probed in this context. This necessarily restricts α to a very small range which is precisely what we require to obtain a small deviation from the Schwarzschild geometry. Thus, under this string inspired assumption of the existence of a minimum length scale l, Eqn. (2.6) indeed is valid as $\alpha \to 0$. In the series expansion of Eqn. (2.5) in powers of α, the last term in Eqn. (2.6) is physically the most relevant term that captures the effect of a small Guass-Bonnet correction to the Schwarzschild geometry. Hence, in the rest of this paper, we shall use the metric in Eqn. (2.4), with the $f(r)$ given by Eqn. (2.6) to analyze the asymptotic quasinormal modes of the d dimensional Schwarzschild black hole in presence of a small Gauss-Bonnet correction.

The black hole described above has a singularity at the origin shielded by a single event horizon [18, 19]. In our approximation, the horizon at $r = r_h$ is determined by the real positive solution of the equation

$$r_h^{2d-4} - 2Mr^{d-1} + 4\alpha M^2 = 0 \tag{2.7}$$

To the leading order in α, $r_h = r_s + \epsilon$, where r_s is the location of the real horizon of the d-dimensional Schwarzschild solution and the correction term $\epsilon \propto -\alpha$. The exact form of the correction term is not important for us and its negative sign is in agreement with the fact that in Gauss-Bonnet black hole the horizon is always located at a magnitude less than that in the case when $\alpha = 0$ [19]. In our approximation, apart from a single real horizon at r_h given above,

there would be in general $(2d-5)$ other fictitious horizons with complex or negative values of r.

3. Asymptotic Quasinormal Modes

In this Section we shall evaluate the asymptotic quasinormal mode frequencies w for the metric given by Eqn. (2.4) with the form of the $f(r)$ given by Eqn. (2.6) in the limit when Im $(w) >>$ Re (w). In the absence of any numerical calculations, at this stage it is an assumption that such modes indeed exist. However, for small values of α this is a reasonable assumption and our final result will also be shown to be consistent with it. We shall consider the quasinormal modes in all space-time dimensions $d \geq 5$ except for $d=6$, as the metric perturbations for the Gauss-Bonnet black hole in $d=6$ are known to be unstable [24]. Below we shall closely follow the monodromy method of ref. [6] and [9].

The asymptotic quasinormal modes for the pure d dimensional Schwarzschild black hole are found by solving a Schrödinger like equation with the Ishibashi-Kodama master potential [25]. In presence of the Gauss-Bonnet term, the tensorial perturbations describing the quasinormal modes still follow a Schrödinger like equation, but with a different potential [24]. In this case, the Schrödinger like equation is given by

$$\left[-\frac{d^2}{dx^2}+V[r(x)]\right]\Phi(x)=\omega^2\Phi(x), \tag{3.1}$$

where x is the tortoise coordinate defined by $dx=\int\frac{dr}{f(r)}$ and the potential V is given by [24]

$$V(r)=q(r)+\left(f\frac{d}{dr}\,ln\,(K)\right)^2+f\frac{d}{dr}\left(f\frac{d}{dr}\,ln\,(K)\right) \tag{3.2}$$

with $K(r)$ and $q(r)$ being given by

$$K(r) = r^{\frac{d-4}{2}}\sqrt{r^2+\frac{\alpha}{(d-3)}\left[(d-5)(1-f(r))-rf'\right]} \tag{3.3}$$

$$q(r) = \left(\frac{f(2-\gamma)}{r^2}\right)\left(\frac{(1-\alpha f''(r))r^2+\alpha(d-5)[(d-6)(1-f(r))-2rf'(r)]}{r^2+\alpha(d-4)[(d-5)(1-f(r))-rf'(r)]}\right), \tag{3.4}$$

where $\gamma=-l(l+d-3)+2$ and $l=2,3,4,\cdots$ The potential in Eqn. (3.2) reduces to the standard Ishibashi-Kodama master potential for the Schwarzschild black hole when $\alpha=0$.

If $\Phi(x)$ describes the quasinormal modes, then in terms of the tortoise coordinate x it must satisfy the boundary conditions

$$\Phi(x) \sim e^{i\omega x} \text{ as } x\to-\infty, \tag{3.5}$$

$$\Phi(x) \sim e^{-i\omega x} \text{ as } x\to+\infty. \tag{3.6}$$

We shall use Eqns. (3.1-3.4) and the boundary conditions given above together with the metric given by Eqn. (2.4) to obtain the highly damped asymptotic quasinormal modes for the d dimensional Schwarzschild black hole with the Gauss-Bonnet correction.

Following ref. [6], we consider the Eqn. (3.1) extended to the whole complex plane. For small values of r, i.e. in the neighbourhood of $r = l$ where l is arbitrarily small, the tortoise coordinate has the form

$$x \sim \frac{r^{2d-3}}{4\alpha M^2(2d-3)}. \tag{3.7}$$

In the same region, the leading singular term of the potential is of the form

$$\begin{aligned} V(r[x]) &= [-32 + \frac{16}{(d-3)^2} + \frac{48}{(d-3)} + 40d - \frac{32d}{(d-3)^2} \\ &\quad - \frac{96d}{(d-3)} - 12d^2 + \frac{16d^2}{(d-3)^2} + \frac{48d^2}{(d-3)}]\frac{1}{16x^2(2d-3)^2} \end{aligned} \tag{3.8}$$

Following [6], the potential can be written as

$$V(r[x]) = \frac{j^2-1}{4x^2}$$

where $j = \frac{(d-1)(d^2+6d-23)^{\frac{1}{2}}}{(2d^2-9d+9)}$. Let us here note that for the Schwarzschild metric in d space-time dimensions, the value of $j = 0$ [6], which is very different from the Gauss-Bonnet case under consideration. We shall make further remarks about this point later in the paper.

For highly damped asymptotic quasinormal modes, we take the frequency w to be approximately purely imaginary. Thus, for the Stokes line defined by Im $(wx) = 0$, we see that x is approximately purely imaginary. This together with Eqn. (3.7) implies that for small r, we have $(4d-6)$ Stokes lines labeled by $n = 0,\ 1,\ 2, \cdots, 4d-7$. The signs of (wx) on these lines are given by $(-1)^{(n+1)}$ and near the origin, the Stokes lines are equispaced by an angle $\frac{\pi}{2d-3}$. Also note that near infinity, $x \sim r$ and Re $(x) = 0$ and Re $(r) = 0$ are approximately parallel. Thus two of the Stokes lines are unbounded and go to infinity. In the limit of small α, just as in the case of Schwarzschild black hole, two more Stokes lines starting from the origin would form a closed loop encircling the real horizon in the complex r plane [9]. The rest of the Stokes lines too would have a structure qualitatively similar to those in the Schwarzschild case.

We now proceed with the calculation of the asymptotic quasinormal modes. The solution of the wave Eqn. (3.1) is given by

$$\Phi(x) = A\sqrt{2\pi\omega x}J_{\frac{j}{2}}(\omega x) + B\sqrt{2\pi\omega x}J_{-\frac{j}{2}}(\omega x), \tag{3.9}$$

where J_ν is the Bessel function of first kind and $A,\ B$ are constants. Since we are considering the situation where Im $(w) \to \infty$, we can use the asymptotic expansion of the Bessel function to write the solution (3.9) as

$$\Phi(x) = \left(Ae^{-i\alpha_+} + Be^{-i\alpha_-}\right)e^{i\omega x} + \left(Ae^{i\alpha_+} + Be^{i\alpha_-}\right)e^{-i\omega x}, \tag{3.10}$$

where $\alpha_\pm = \frac{\pi}{4}(1 \pm j)$. Now consider a point z_- near $r \sim \infty$ and situated on one of the unbounded Stokes lines which is asymptotically parallel to the negative imaginary axis in the complex r plane. On such a point z_-, we have $wx \to \infty$ and thus the asymptotic form of the solution (3.10) is valid. Imposing the boundary condition (3.6) we get from (3.10) that

$$Ae^{-i\alpha_+} + Be^{-i\alpha_-} = 0. \tag{3.11}$$

Consider now a point z_+ again near $r \sim \infty$ and situated on one of the unbounded Stokes lines which is asymptotically parallel to the positive imaginary axis in the complex r plane. From the geometry of the Stokes lines and their equispaced nature near the origin, it is easy to see that in order to pass from the point z_- to z_+ while always staying on the Stokes lines, it is necessary to traverse an angle $\frac{3\pi}{2d-3}$ in the complex r plane, which amounts to a 3π rotation in the tortoise coordinate x. Using the analytic continuation formula for the Bessel function

$$\sqrt{2\pi e^{3\pi i}\omega x}\; J_{\pm\frac{j}{2}}\left(e^{3\pi i}\omega x\right) = e^{\frac{3\pi i}{2}(1\pm j)}\sqrt{2\pi\omega x}\; J_{\pm\frac{j}{2}}\left(\omega x\right), \tag{3.12}$$

the solution at the point z_+ can be written as

$$\Phi(x) = \left(Ae^{7i\alpha_+} + Be^{7i\alpha_-}\right)e^{i\omega x} + \left(Ae^{5i\alpha_+} + Be^{5i\alpha_-}\right)e^{-i\omega x}. \tag{3.13}$$

We now close the two asymptotic branches of the Stokes lines by a contour along $r \sim \infty$ on which Re $(x) > 0$. Since we are considering modes with Im $(w) \to \infty$, on this part of the contour e^{iwx} is exponentially small. Thus, for the purpose of monodromy calculation, we rely only on the coefficient of e^{-iwx} [6]. As the contour is completed this coefficient picks up a multiplicative factor given by

$$\frac{Ae^{5i\alpha_+} + Be^{5i\alpha_-}}{Ae^{i\alpha_+} + Be^{i\alpha_-}}. \tag{3.14}$$

The monodromy of e^{-iwx} along this clockwise contour is $e^{-\frac{\pi\omega}{k}}$ where $k = \frac{1}{2}f'(r_h)$ is the surface gravity at the Gauss-Bonnet real horizon r_h. Thus the complete monodromy of the solution to the wave equation along this clockwise contour is

$$\frac{Ae^{5i\alpha_+} + Be^{5i\alpha_-}}{Ae^{i\alpha_+} + Be^{i\alpha_-}}e^{-\frac{\pi\omega}{k}}. \tag{3.15}$$

The contour discussed above can now be smoothly deformed to a small circle going clockwise around the horizon at $r = r_h$. Near $r = r_h$ the potential in the wave equation approximately vanishes. From the boundary condition (3.5), we see that the solution of the wave equation (3.1) near the black hole event horizon is of the form

$$\Phi(x) \sim Ce^{i\omega x} \tag{3.16}$$

where C is a constant. The monodromy of Φ going around the small clockwise circle around the event horizon is thus given by $e^{\frac{\pi\omega}{k}}$. Since the two contours are homotopic, the monodromies around them are equal. Thus, from Eqn. (3.15) and Eqn. (3.16) we have

$$\frac{Ae^{5i\alpha_+} + Be^{5i\alpha_-}}{Ae^{i\alpha_+} + Be^{i\alpha_-}}e^{-\frac{\pi\omega}{k}} = e^{\frac{\pi\omega}{k}}. \tag{3.17}$$

Eliminating the constants A and B from Eqn. (3.11) and Eqn. (3.17) , we get

$$\mathrm{e}^{\frac{2\pi w}{k}} = -\frac{\sin(\frac{3\pi}{2})j}{\sin(\frac{\pi}{2})j}. \tag{3.18}$$

Equivalently, we have,

$$w = T_H \log\left|\frac{\sin(\frac{3\pi}{2})j}{\sin(\frac{\pi}{2})j}\right| + 2\pi i T_H\left(n+\frac{1}{2}\right), \tag{3.19}$$

where $T_H = \frac{k}{2\pi}$ is the Hawking temperature of the black hole (in units of $\hbar = 1$.) Eqn. (3.19) with $j = \frac{(d-1)(d^2+6d-23)^{\frac{1}{2}}}{(2d^2-9d+9)}$ provides an analytic expression for the highly damped asymptotic quasinormal modes of the Gauss-Bonnet black hole in the limit where the Gauss-Bonnet coupling constant α is small. In the next Section, we discuss some of the features and implications of the result obtained above.

4. Discussion

In this Paper we have calculated the asymptotic quasinormal frequencies of the d dimensional Schwarzschild black hole in presence of a small Gauss-Bonnet correction term, where we have used the string inspired assumption of the existence of a minimum fundamental length scale l. Our analysis is relevant only when the Gauss-Bonnet coupling α is suitably restricted such that $8\alpha M << l^{d-1}$. For a massive black hole, this condition restricts α to very small values. This implies that the geometry that we are considering is very close to the Schwarzschild background with the Gauss-Bonnet term providing a small modification of the Schwarzschild geometry.

The quasi-normal mode frequencies are functions of the spacetime dimension d and they also depend on the metric parameters M and α through the surface gravity k. It may be noted that if the limit $\alpha \to 0$ is taken after the calculation of the asymptotic quasinormal mode, then the resulting quasinormal frequencies do not tend towards those for the Schwarzschild black hole for which $j = 0$. The reason for this is not difficult to understand. The point is that in the calculation of the asymptotic quasinormal mode, the short distance singularity structure of the potential in (3.2) plays a crucial role [6]. In presence of a small but nonzero value of the Gauss-Bonnet coupling α, the coefficient of the leading short distance singularity is very different from that when $\alpha = 0$. Thus, in the limit when $\alpha \to 0$, even though the surface gravity for the Gauss-Bonnet black hole smoothly goes over to that for the Schwarzschild case, the asymptotic quasinormal frequencies do not. The situation here is very similar to the well known fact that the asymptotic quasinormal frequencies of the Reissner-Nordström black hole do not tend to those for the Schwarzschild in the limit where the charge is taken to zero [6, 9]. Thus, if the Schwarzschild black hole is considered to be the zero charge limit of the Reissner-Nordström geometry, and if the Schwarzschild quasinormal frequencies are to be obtained from analyzing the Reissner-Nordström system, then the charge equal to zero limit in the Reissner-Nordström metric has to be taken before the calculation of the quasinormal frequencies. It may also be noted that approximations in the metric which change the nature of the singularity structure

of the differential equation have been used previously in the literature as well for the purpose of making of quasinormal modes estimations [26].

In contrast to the Schwarzschild case, the real part of the asymptotic quasinormal frequencies associated with the tensorial perturbations of the Gauss-Bonnet black hole for small values of α do not exhibit any universality with respect to the number of space-time dimensions and is unlikely to have that feature for other type of perturbations as well. However, as the number of dimensions is increased, the real part of the quasinormal frequencies tend towards zero. The conjecture of Hod [13] therefore does not seem to be valid in presence of a small but finite Gauss-Bonnet coupling α. Our result thus provides a clear distinction between Einstein gravity and its string derived variant.

The metric given by $f(r)$ in Eqn. (2.6) certainly does not capture the full effect of the general Gauss-Bonnet term for arbitrary values of the coupling α. It would be nice to obtain an analytic expression for the asymptotic quasinormal frequencies of the Gauss-Bonnet black hole for a general value of the coupling α, which has so far not been possible. It would also be very important to obtain the quasinormal frequencies of this system numerically. The asymptotic quasinormal frequencies for related string derived models of gravity, especially those with cosmological constant and any associated connection to the dual conformal field theories would also be interesting to find. Some of this work is currently under progress [27].

Acknowledgment

The authors wish to thank I. P. Neupane for useful comments.

References

[1] T. Regge and J. A. Wheeler, Phys. Rev. **108**, (1957) 1063.

[2] C. V. Vishveshwara, Phys. Rev. **D1**, (1970) 2870; Nature **227**, (1970) 936.

[3] K. D. Kokkotas and B. G. Schmidt, Living Rev. Rel. **2**, (1999) 2; H-P. Nollert, Class. Quant. Grav. **16**, (1999) R159; K. D. Kokkotas and N. Stergioulas, gr-qc/0506083.

[4] A. Barrau, J. Grain and S. O. Alexeyev, Phys. Lett. **B584**, (2004) 114.

[5] S. Chandrasekhar and S. Detweiler, Proc. Roy. Soc. (London), **A343**, (1975) 289; V. Ferrari and B. Mashhoon, Phys. Rev. **D30**, (1984) 295.

[6] L. Motl and A. Neitzke, Adv. Theor. Math. Phys. **7**, (2003) 307.

[7] S. Das and S. Shankaranarayanan, Class. Quant. Grav. **22**, (2005) L7.

[8] V. Cardoso, J. Natário and R. Schiappa, J. Math. Phys. **45**, (2004) 4698.

[9] J. Natário and R. Schiappa, hep-th/0411267.

[10] R. G. Daghigh and G. Kunstatter, gr-qc/0505044; gr-qc/0507019.

[11] D. Birmingham and S. Carlip, Phys. Rev. Lett. **92**, (2004) 111302.

[12] Kumar S. Gupta and Siddhartha Sen, Phys. Lett. **B618**, (2005) 237.

[13] S. Hod, Phys. Rev. Lett. **81**, (1998) 4293.

[14] J. D. Bekenstein and V. F. Mukhanov, Phys. Lett. **B360**, (1995) 7.

[15] M. R. Setare, Class. Quant. Grav. **21**, (2004) 1453; ibid, Phys. Rev. **D69**, (2004) 044016.

[16] O. Dreyer, Phys. Rev. Lett. **90**, (2003) 081301.

[17] D. Birmingham, I. Sachs and S. N. Solodukhin, Phys. Rev. Lett. **88**, (2002) 151301; D. Birmingham, I. Sachs and S. N. Solodukhin, Phys. Rev. **D67**, (2003) 104026; K. Krasnov and S. N. Solodukhin, Adv. Theor. Math. Phys. **8**, (2004) 421.

[18] David G. Boulware and S. Deser, Phys. Rev. Lett. **55**, (1985) 2656.

[19] D. L. Wiltshire, Phys. Rev. **D38**, (1988) 2445.

[20] B. Zwiebach, Phys. Lett **B156**, (1985) 315.

[21] K. A. Meissner and O. Olechowski, Phys. Rev. **D65**, (2002) 064017.

[22] M. Cvetic, S. Nojiri and S. D. Odintsov, Nucl. Phys. **B625**, (2002) 295; S. Nojiri, S. D. Odintsov and S. Ogushi, Phys. Rev. **D65**, (2002) 023521; Y. M. Cho and I. P. Neupane, Phys. Rev. **D66**, (2002) 024044; I. P. Neupane, Phys. Rev. **D67**, (2003) 061501; T. Clunan, S. F. Ross and D. J. Smith, Class. Quant. Grav. **21**, (2004) 3447.

[23] R. Konoplya, Phys. Rev. **D71**, (2005) 024038.

[24] G. Dotti and R. J. Gleiser, Class. Quant. Grav. **22**, (2005) L1.

[25] V. Cardoso, O. J. C. Dias and J. P. S. Lemos, Phys. Rev. **D67**, (2003) 064026; A. Ishibashi and H. Kodama, Prog. Theor. Phys. **110**, (2003) 701; A. Ishibashi and H. Kodama, Prog. Theor. Phys. **110**, (2003) 901.

[26] G. Siopsis, Phys. Lett. **B590**, (2004) 105.

[27] Sayan K. Chakrabarti and Kumar S. Gupta (in preparation).